WALS

Hiroshima · 10-12 NOV 2025

Analyzing the structure of a mathematics sequence in a Japanese fourth-grade textbook

Bansho, *kyouzai kenkyū*, and noticing

Edon Gashi, Stéphane Clivaz

Lausanne University of Teacher Education, Switzerland
Lausanne Laboratory of Lesson Study

3LS hep/

Outline

1. Introduction – In the spirit of Tsunagu
2. What's in a Japanese math textbook?
3. Insights and implications for LS
4. Conclusion

Introduction – In the spirit of Tsunagu

Introduction – In the spirit of Tsunagu

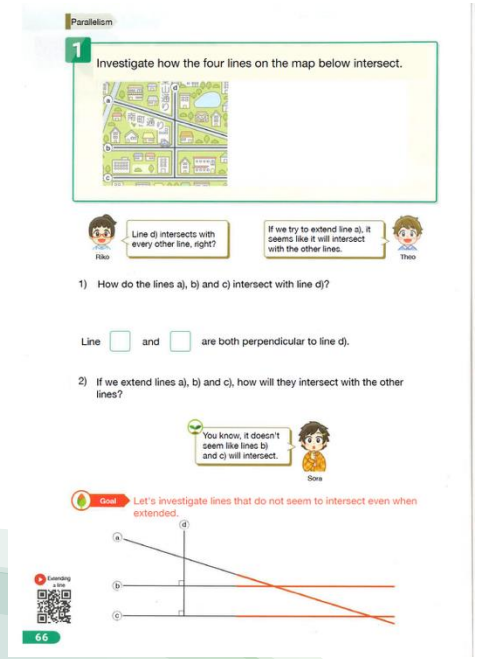
- This year's theme is Tsunagu (繋ぐ), 'to connect'.
- Different cultures have different instructional materials.
- Let's find out what's in a Japanese mathematics textbook.

What’s in a Japanese math textbook?

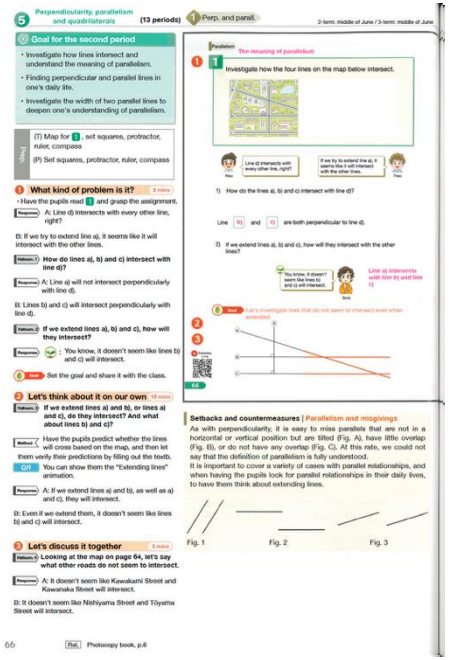
What's in a Japanese math textbook?

- *wakuwaku suugaku*, fourth grade.
- Integrated collection of instructional materials.
- We chose to focus on a sequence regarding perpendicular and parallel (straight) lines.

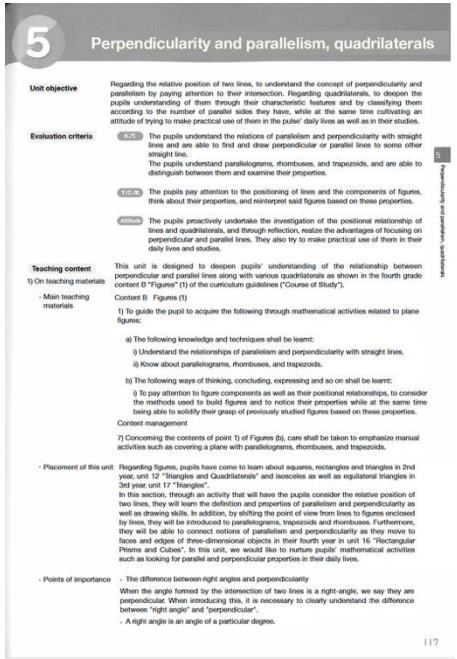
What's in a Japanese math textbook?



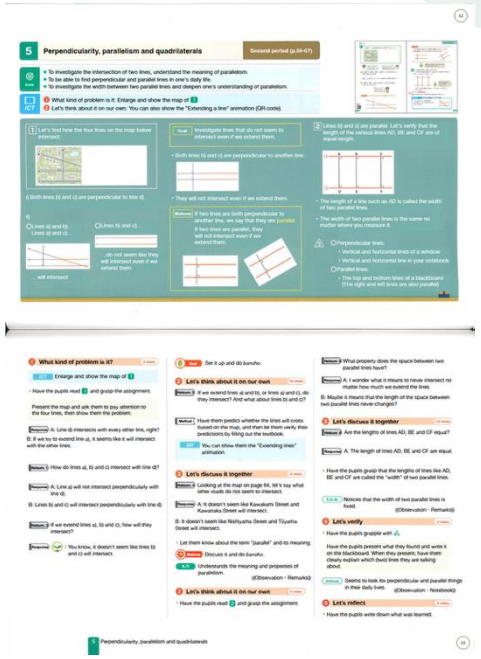
Pupil textbook



Teacher textbook



Research guidebook



Bansho book

Hiroshima · 10-12 NOV 2025

1 Investigate how the four lines on the map below intersect.

Riko

Line d) intersects with every other line, right?

If we try to extend line a), it seems like it will intersect with the other lines.

Theo

1) How do the lines a), b) and c) intersect with line d)?

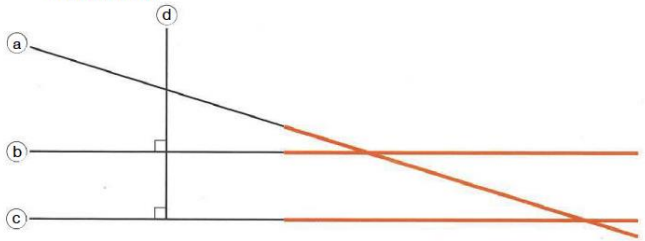
Line ☐ and ☐ are both perpendicular to line d).

2) If we extend lines a), b) and c), how will they intersect with the other lines?

You know, it doesn't seem like lines b) and c) will intersect.

Sora

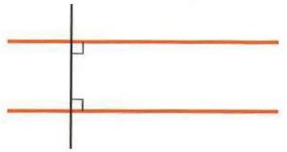
Goal Let's investigate lines that do not seem to intersect even when extended.



Extending a line

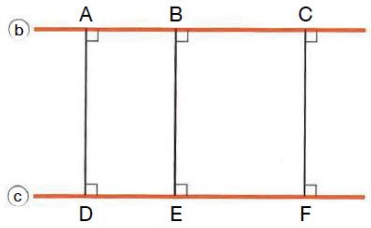
Matome

Two lines that are perpendicular to another line are called parallel.
Parallel lines never intersect no matter how much we extend them.



We do not call parallel two curved lines that never intersect.

2 On the figure below, lines b) and c) are parallel. Let's verify that the lengths of each line AD, BE and CF are the same.



We call lengths like the one from line AD and so on the width of two parallel lines.

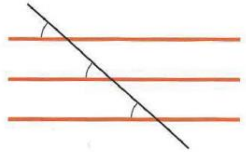
The width of two parallel lines is the same no matter where you measure it.

Learning made alive Let's search for it!

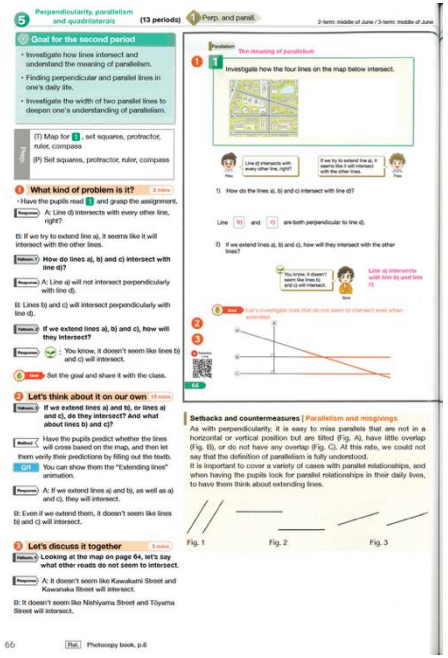
3 Let's find parallel and perpendicular lines in our daily lives.

Math pocket **Angles we can make from parallel lines**

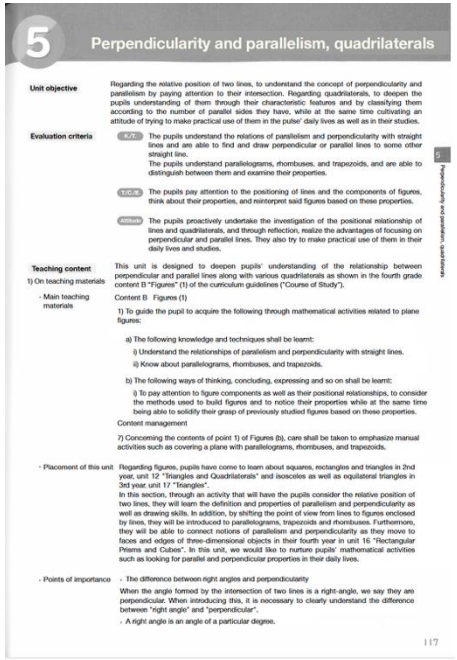
Like the figure on the right, if you draw a line through parallel lines, the angles formed are all of equal size.



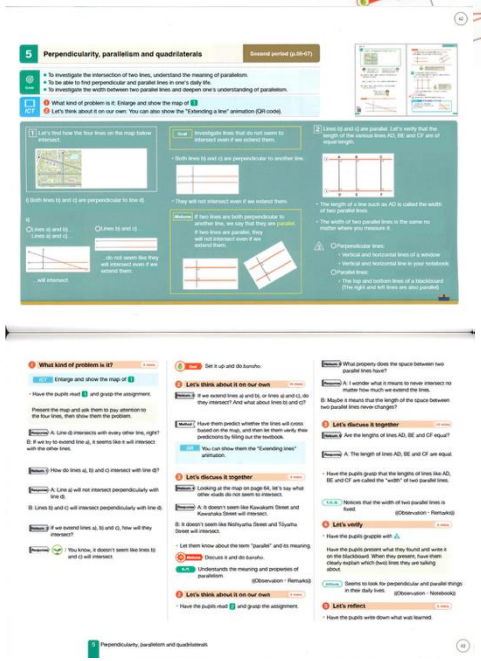
What's in a Japanese math textbook?



Teacher textbook



Research guidebook



Bansho book

Hiroshima · 10-12 NOV 2025

5 Perpendicularity, parallelism and quadrilaterals (13 periods)

1 Perp. and parall. 2-term: middle of June / 3-term: middle of June

Goal for the second period

- Investigate how lines intersect and understand the meaning of parallelism.
- Finding perpendicular and parallel lines in one's daily life.
- Investigate the width of two parallel lines to deepen one's understanding of parallelism.

Prep.

(T) Map for 1, set squares, protractor, ruler, compass

(P) Set squares, protractor, ruler, compass

1 What kind of problem is it? 5 mins

- Have the pupils read 1 and grasp the assignment.

Response A: Line d) intersects with every other line, right?

B: If we try to extend line a), it seems like it will intersect with the other lines.

Hatsum. 1 **How do lines a), b) and c) intersect with line d)?**

Response A: Line a) will not intersect perpendicularly with line d).

B: Lines b) and c) will intersect perpendicularly with line d).

Hatsum. 2 **If we extend lines a), b) and c), how will they intersect?**

Response : You know, it doesn't seem like lines b) and c) will intersect.

Goal Set the goal and share it with the class.

2 Let's think about it on our own 10 mins

Hatsum. 3 **If we extend lines a) and b), or lines a) and c), do they intersect? And what about lines b) and c)?**

Method Have the pupils predict whether the lines will cross based on the map, and then let them verify their predictions by filling out the textb.

QR You can show them the "Extending lines" animation.

Response A: If we extend lines a) and b), as well as a) and c), they will intersect.

B: Even if we extend them, it doesn't seem like lines b) and c) will intersect.

3 Let's discuss it together 5 mins

Hatsum. 4 **Looking at the map on page 64, let's say what other roads do not seem to intersect.**

Response A: It doesn't seem like Kawakami Street and Kawanaka Street will intersect.

B: It doesn't seem like Nishiyama Street and Tōyama Street will intersect.

Parallelism

The meaning of parallelism

1 Investigate how the four lines on the map below intersect.

Riko: Line d) intersects with every other line, right?

Theo: If we try to extend line a), it seems like it will intersect with the other lines.

1) How do the lines a), b) and c) intersect with line d)?

Line b) and c) are both perpendicular to line d).

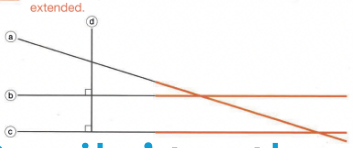
2) If we extend lines a), b) and c), how will they intersect with the other lines?

Sora: You know, it doesn't seem like lines b) and c) will intersect.

Line a) intersects with line b) and line c)

Goal Let's investigate lines that do not seem to intersect even when extended.

2 3



Extending a line

66

Pupils' textbook

5 Perpendicularity and parallelism, quadrilaterals

Matome

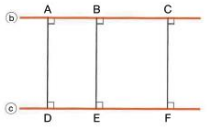
Two lines that are perpendicular to another line are called parallel.
Parallel lines never intersect no matter how much we extend them.

We do not call parallel two curved lines that never intersect.

The distance between parallel lines (width)

2 On the figure below, lines a) and c) are parallel. Let's verify that the lengths of each line AD, BE and CF are the same.

3



We call lengths like the one from line AD and so on the width of two parallel lines.

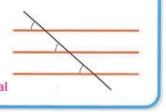
The width of two parallel lines is the same no matter where you measure it.

Learning made alive Let's search for it!

4 3 Let's find parallel and perpendicular lines in our daily lives. Omitted.

Math pocket **Angles we can make from parallel lines**

Like the figure on the right, if you draw a line through parallel lines, the angles formed are all of equal size.



Corresponding angles of a diagonal are equal

5 Learned in the second grade of middle school.

67

Setbacks and countermeasures | Parallelism and misgivings

As with perpendicularity, it is easy to miss parallels that are not in a horizontal or vertical position but are tilted (Fig. A), have little overlap (Fig. B), or do not have any overlap (Fig. C). At this rate, we could not say that the definition of parallelism is fully understood.
It is important to cover a variety of cases with parallel relationships, and when having the pupils look for parallel relationships in their daily lives, to have them think about extending lines.

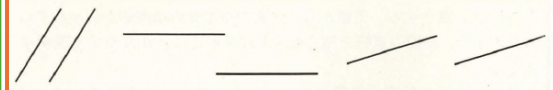


Fig. 1 Fig. 2 Fig. 3

Common difficulties

Reference **The definition and properties of parallelism**

There are three ways to define parallel lines lying in the same plane.

1) No matter how far one stretches the two lines, they never intersect.
2) Two lines are parallel if the width between them is always the same.
3) Two perpendicular lines to another that intersect said line at the same angle are said to be parallel.

For 1), it is not actually possible to stretch a line infinitely to check whether they are parallel. It also doesn't lead to any construction method.
For 2), the word "width" is a complex concept for pupils, and in order to verify that the "width" is the same, it is necessary to draw a perpendicular line between two lines to confirm that the distance between them is constant.
As such, if we treat 3) as the definition, 1) and 2) are properties. Considering the developmental stage of the pupils, we choose to use definition that "two lines perpendicular to another line are parallel".

▶ The distance between parallel lines (width) ⇒ Research guidebook p.122

Reference to the research guidebook

- Let them know the term "parallel" and its meaning.

Warning Make the pupils aware that two non-straight lines that do no meet when extended are not called parallel.

Matome Discuss (*matome*) and share with the class.

K./T. Understands the meaning and properties of parallelism. ((Observation・Notebook))

2 Let's think about it on our own 5 mins

- Have the pupils read 2 and grasp the assignment.

Hatsum. 5 **What property does the space between two parallel lines have?**

Response A: I wonder what it means to never intersect no matter how much we extend the lines.

B: Maybe it means that the length of the space between two parallel lines never changes?

3 Let's discuss it together 10 mins

- Have the pupils grapple with 2, and present their findings.

Hatsum. 6 **Are the lengths of lines AD, BE and CF equal?**

Method Have the pupils investigate using a compass and a ruler.

Response A: The length of lines AD, BE and CF are equal.

- Have the pupils notice that when comparing lengths, using a compass is very useful.
- Stress the fact that lengths such as AD, BE and CF are called the "width" of two parallel lines.
- Summarize what the students found. (*matome*)

T./C./E. Notices that the width of two parallel lines is fixed. ((Observation・Remarks))

4 Let's verify 5 mins

- Have the pupils grapple with 3.

Attitude Seems to look for perpendicular and parallel things in their daily lives. ((Observation・Notebook))

Depending on the state of the pupils, you can introduce the math pocket to them.

5 Let's reflect 5 mins

- Have the pupils write down what was learned.

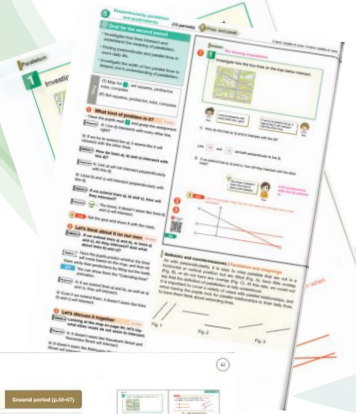
5 Perpendicularity and parallelism, quadrilaterals

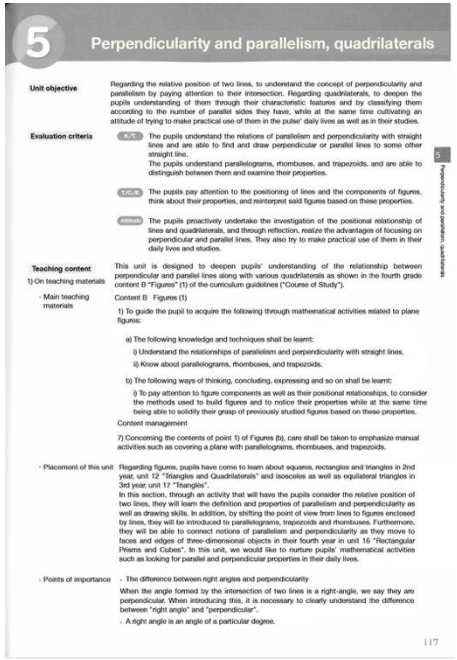
Comments

Hiroshima・10

nep/

What's in a Japanese math textbook?




5 Perpendicularity and parallelism, quadrilaterals

Unit objective

Regarding the relative position of two lines, to understand the concept of perpendicularity and parallelism by paying attention to their intersection. Regarding quadrilaterals, to deepen the pupils understanding of them through their characteristic features and by classifying them according to the number of parallel sides they have, while at the same time cultivating an attitude of trying to make practical use of them in the pulse' daily lives as well as in their studies.

Evaluation criteria

A/K The pupils understand the relations of parallelism and perpendicularity with straight lines and are able to find and draw perpendicular or parallel lines to some other straight line.
The pupils understand parallelograms, rhombuses, and trapezoids, and are able to distinguish between them and examine their properties.

T/C/E The pupils pay attention to the positioning of lines and the components of figures, think about their properties, and reinterpret said figures based on these properties.

Attitude The pupils proactively undertake the investigation of the positional relationship of lines and quadrilaterals, and through reflection, realize the advantages of focusing on perpendicular and parallel lines. They also try to make practical use of them in their daily lives and studies.

Teaching content

This unit is designed to deepen pupils' understanding of the relationship between perpendicular and parallel lines along with various quadrilaterals as shown in the fourth grade content B "Figures" (1) of the curriculum guidelines ("Course of Study").

1) On teaching materials

- Main teaching materials

Content B Figures (1)

1) To guide the pupil to acquire the following through mathematical activities related to plane figures:

a) The following knowledge and techniques shall be learnt:

i) Understand the relationships of parallelism and perpendicularity with straight lines.

ii) Know about parallelograms, rhombuses, and trapezoids.

b) The following ways of thinking, concluding, expressing and so on shall be learnt:

i) To pay attention to figure components as well as their positional relationships, to consider the methods used to build figures and to notice their properties while at the same time being able to solidify their grasp of previously studied figures based on these properties.

Content management

7) Concerning the contents of point 1) of Figures (b), care shall be taken to emphasize manual activities such as covering a plane with parallelograms, rhombuses, and trapezoids.

- Placement of this unit

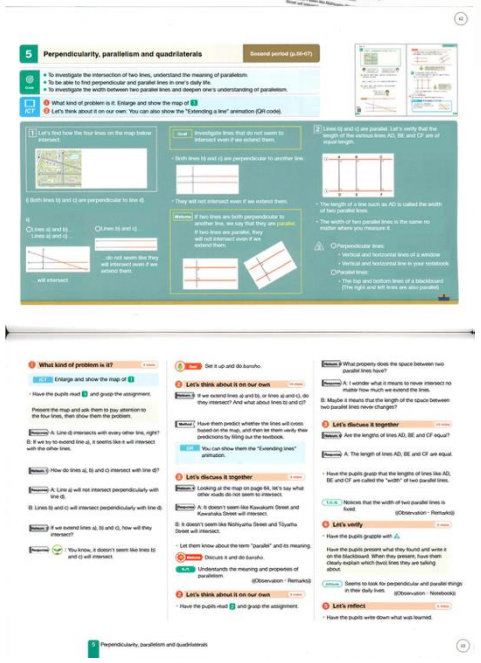
Regarding figures, pupils have come to learn about squares, rectangles and triangles in 2nd year, unit 12 "Triangles and Quadrilaterals" and isosceles as well as equilateral triangles in 3rd year, unit 17 "Triangles".
In this section, through an activity that will have the pupils consider the relative position of two lines, they will learn the definition and properties of parallelism and perpendicularity as well as drawing skills. In addition, by shifting the point of view from lines to figures enclosed by lines, they will be introduced to parallelograms, trapezoids and rhombuses. Furthermore, they will be able to connect notions of parallelism and perpendicularity as they move to faces and edges of three-dimensional objects in their fourth year in unit 16 "Rectangular Prisms and Cubes". In this unit, we would like to nurture pupils' mathematical activities such as looking for parallel and perpendicular properties in their daily lives.

- Points of importance

・The difference between right angles and perpendicularity
When the angle formed by the intersection of two lines is a right-angle, we say they are perpendicular. When introducing this, it is necessary to clearly understand the difference between "right angle" and "perpendicular".
・A right angle is an angle of a particular degree.

117

Research guidebook


5 Perpendicularity, parallelism and quadrilaterals

Bansho book

3S hep/

Objective

Evaluation, instruction

Teaching content, curriculum

5 Perpendicularity and parallelism, quadrilaterals

Unit objective

Regarding the relative position of two lines, to understand the concept of perpendicularity and parallelism by paying attention to their intersection. Regarding quadrilaterals, to deepen the pupils understanding of them through their characteristic features and by classifying them according to the number of parallel sides they have, while at the same time cultivating an attitude of trying to make practical use of them in the pulse' daily lives as well as in their studies.

Evaluation criteria

K./T. The pupils understand the relations of parallelism and perpendicularity with straight lines and are able to find and draw perpendicular or parallel lines to some other straight line.
The pupils understand parallelograms, rhombuses, and trapezoids, and are able to distinguish between them and examine their properties.

T./C./E. The pupils pay attention to the positioning of lines and the components of figures, think about their properties, and reinterpret said figures based on these properties.

Attitude The pupils proactively undertake the investigation of the positional relationship of lines and quadrilaterals, and through reflection, realize the advantages of focusing on perpendicular and parallel lines. They also try to make practical use of them in their daily lives and studies.

Teaching content

1) On teaching materials

This unit is designed to deepen pupils' understanding of the relationship between perpendicular and parallel lines along with various quadrilaterals as shown in the fourth grade content B "Figures" (1) of the curriculum guidelines ("Course of Study").

- Main teaching materials

Content B Figures (1)

1) To guide the pupil to acquire the following through mathematical activities related to plane figures:

a) The following knowledge and techniques shall be learnt:

i) Understand the relationships of parallelism and perpendicularity with straight lines.

ii) Know about parallelograms, rhombuses, and trapezoids.

b) The following ways of thinking, concluding, expressing and so on shall be learnt:

i) To pay attention to figure components as well as their positional relationships, to consider the methods used to build figures and to notice their properties while at the same time being able to solidify their grasp of previously studied figures based on these properties.

Content management

7) Concerning the contents of point 1) of Figures (b), care shall be taken to emphasize manual activities such as covering a plane with parallelograms, rhombuses, and trapezoids.

- Placement of this unit

Regarding figures, pupils have come to learn about squares, rectangles and triangles in 2nd year, unit 12 "Triangles and Quadrilaterals" and isosceles as well as equilateral triangles in 3rd year, unit 17 "Triangles".
In this section, through an activity that will have the pupils consider the relative position of two lines, they will learn the definition and properties of parallelism and perpendicularity as well as drawing skills. In addition, by shifting the point of view from lines to figures enclosed by lines, they will be introduced to parallelograms, trapezoids and rhombuses. Furthermore, they will be able to connect notions of parallelism and perpendicularity as they move to faces and edges of three-dimensional objects in their fourth year in unit 16 "Rectangular Prisms and Cubes". In this unit, we would like to nurture pupils' mathematical activities such as looking for parallel and perpendicular properties in their daily lives.

- Points of importance

- The difference between right angles and perpendicularity

When the angle formed by the intersection of two lines is a right-angle, we say they are perpendicular. When introducing this, it is necessary to clearly understand the difference between "right angle" and "perpendicular".

- A right angle is an angle of a particular degree.

- Perpendicularity is a relationship between two lines.

In other words, a right angle is a notion related to figures, perpendicularity is a notion related to the relative position of two lines. In general, relational concepts such as parallelism, perpendicularity and congruency play an important role when the focus is on analyzing figures. In particular, parallelism and perpendicularity become crucial viewpoints when looking at characteristics of basic figures.

- Regarding perpendicularity and parallelism

Among the various ways of intersecting two lines, have pupils grasp perpendicularity as the positional relationship of two straight lines intersecting at a right angle. In view of future studies of figures, it is important to ensure that the pupils grasp perpendicular relations when the entire figure is tilted, or when there is no intersection.
While parallelism could be defined as "two straight lines (on the same plane) that do not intersect" or "straight lines with equal width in between", here in order to make it easier to understand its relationship to perpendicularity, we use drawing to define it as "two lines perpendicular to another line".

Mathematical analysis

2) On the pupils' viewpoint

Up until now, the pupils have engaged in mathematical activities where they focused on counting sides and vertices of triangles and quadrilaterals, as well as their right angles and the lengths of their sides, while also examining their distinguishing features and component parts. In this unit, the pupils will distinguish between various quadrilaterals by focusing on parallelism which they will study in the second period. In doing so, we want the pupils to become aware of the definition and properties of parallelism.
Further, the diagonals introduced in the 10th period are also necessary components for the sum of the angles of quadrilaterals and polygons in grade 5, unit 7 "congruency of figures" and in the area of parallelograms in grade 5, unit 10 "area", among other things.

Pupils' viewpoint

3) On devising instruction and evaluation

- Devising instruction

In order for the pupils not to end up learning only by memorizing the meaning of terminology and figure methods, we want their understanding to occur by way of mathematical activities. In addition, when they work on finding parallel and perpendicular lines in their daily lives, it is important to make the pupils genuinely feel that these do actually occur in various daily life activities. Further, in order for the pupils to consolidate their skills in drawing parallel and perpendicular lines as well as parallelograms, make sure that they accumulate a lot of experience in working with a triangular ruler and a compass.

- Devising the evaluation

Knowledge on parallelism and perpendicularity, as well as the pupils' understanding of various properties of quadrilaterals, will be evaluated based on their notebook and remarks. Technical skill will be evaluated on their ability to draw parallel and perpendicular lines, as well as whether they are able to draw various quadrilateral figures, based on their notebook and printouts. Mathematical opinions and thinking will be evaluated, from their notebook and remarks, on whether they are thinking of the relation of parallel and perpendicular lines and of the properties of various quadrilaterals. Attitude will be evaluated based on whether they are trying to find perpendicular and parallel lines, as well as various quadrilaterals, in their surroundings.

Teach./Eval. plan

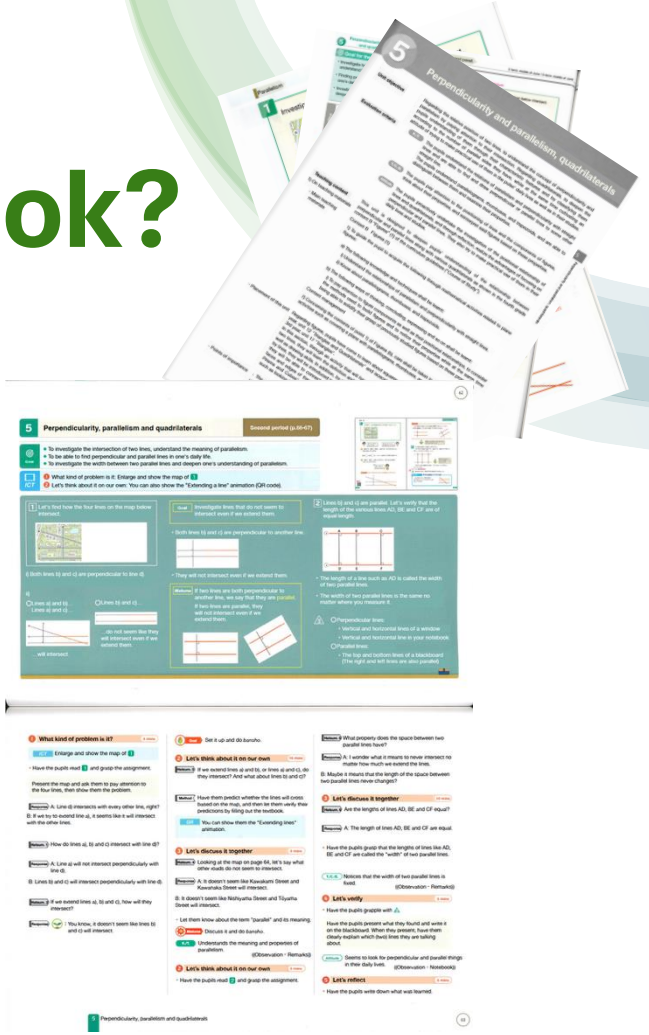
Sub-unit	Period	Pages	Objective	Instruction contents (( )) denotes terminology	Main appraisal criteria (( )) denotes evaluation method
Preparation	—	143	• Verify the pupils' understanding of second-year notions of triangles and quadrilaterals.		
○ Introduction	1	63	• Grasp the theme of this unit (how lines can vary) based on the way road crossings	• Motivate the investigation of line intersections by examinating actual road crossings.	Attitude Shows an interest in the intersection of lines and investigates what kind of things exist. ((Observation・Remarks))
1 Parallelism and perp.		64～65	• Investigate the intersection of two lines and understand the meaning of perpendicularity.	• The relations of two lines and perpendicularity. ((Perp.))	K./T. Understands the significance of perpendicularity and is able to find perp. lines to a given line. ((Observation・Notebooks))

Sequence plan

Hiroshima・10

hep/

What's in a Japanese math textbook?

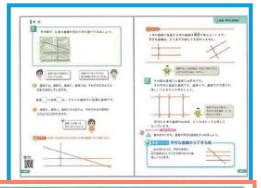


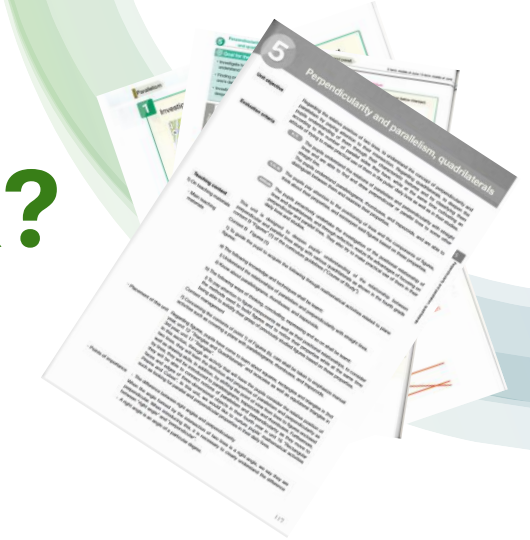
Bansho book

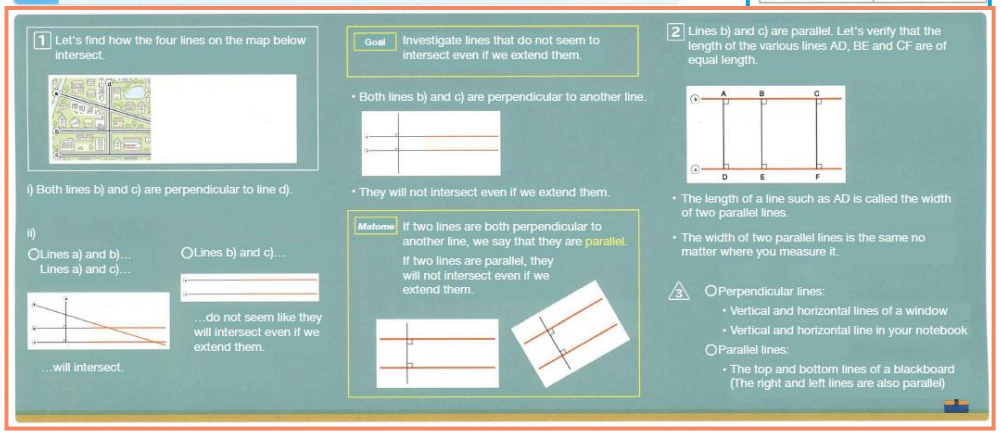
What's in a book?

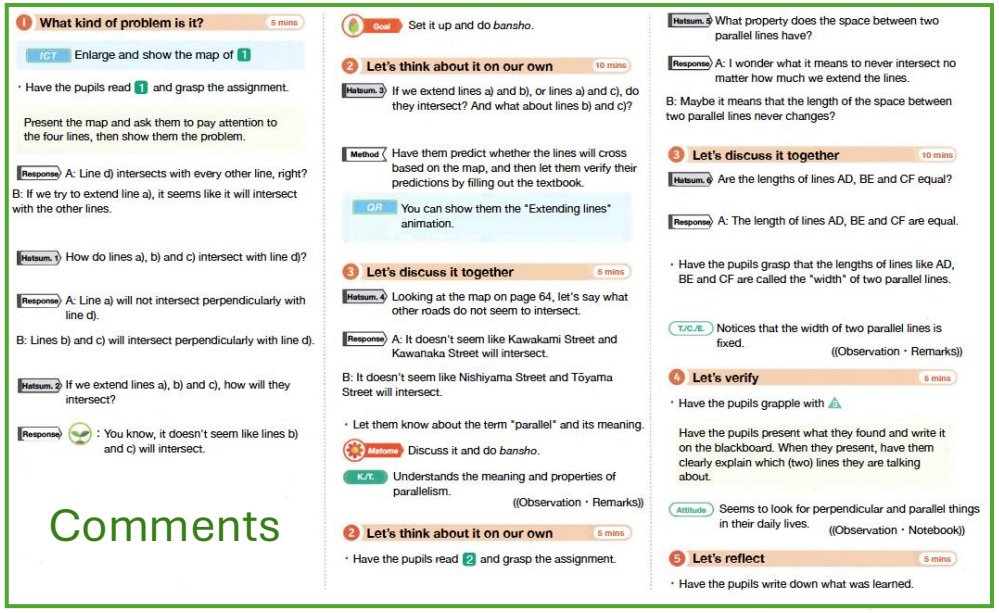
Board organization (bansho)

Pupil's textbook







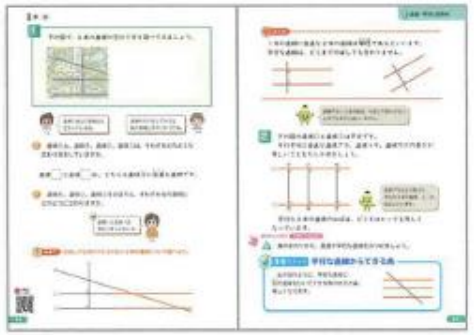


Comments

Hiroshima・10-12 NOV 2025

5 Perpendicularity, parallelism and quadrilaterals

Second period (p.66-67)



Goal

- To investigate the intersection of two lines, understand the meaning of parallelism.
- To be able to find perpendicular and parallel lines in one's daily life.
- To investigate the width between two parallel lines and deepen one's understanding of parallelism.

ICT

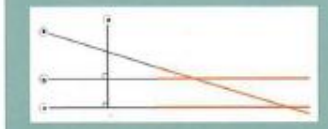
1. What kind of problem is it: Enlarge and show the map of 1
2. Let's think about it on our own: You can also show the "Extending a line" animation (QR code).

1 Let's find how the four lines on the map below intersect.

i) Both lines b) and c) are perpendicular to line d).

ii)

○Lines a) and b)...
Lines a) and c)...



...will intersect.

○Lines b) and c)...

...do not seem like they will intersect even if we extend them.

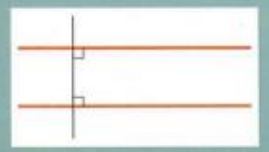
Goal Investigate lines that do not seem to intersect even if we extend them.

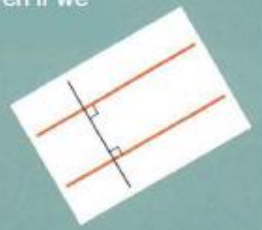
- Both lines b) and c) are perpendicular to another line.

- They will not intersect even if we extend them.

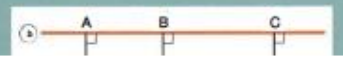
Matome If two lines are both perpendicular to another line, we say that they are parallel.

If two lines are parallel, they will not intersect even if we extend them.





2 Lines b) and c) are parallel. Let's verify that the length of the various lines AD, BE and CF are of equal length.



(The right and left lines are also parallel)

If this interests you, please join us in the bansho workshop!

(day 2, 11:00@Conf. Management Room 6)

Insights and implications for LS

Hiroshima · 10-12 NOV 2025

Insights and implications for LS

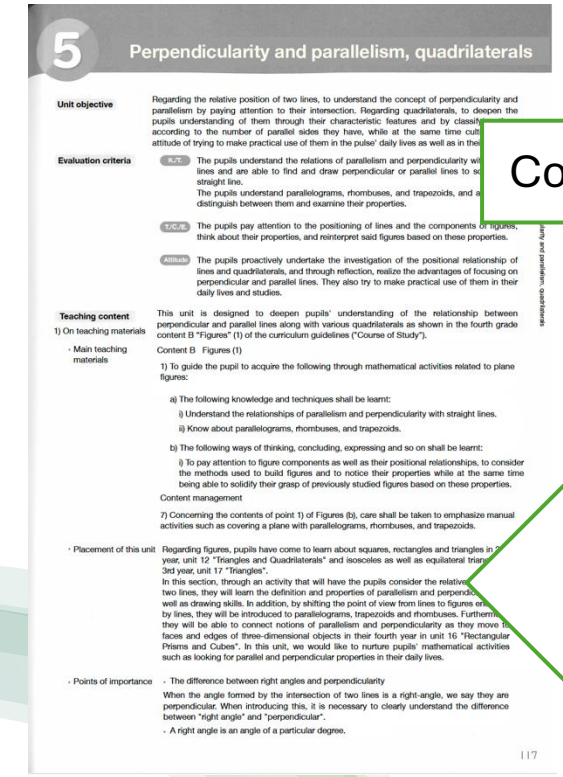
The **research guidebook** gives structure to the sequence, and didactic considerations clarify the mathematics to be taught.

The **teacher textbook** gives structure to the lesson, and provides additional information (e.g., student difficulties).

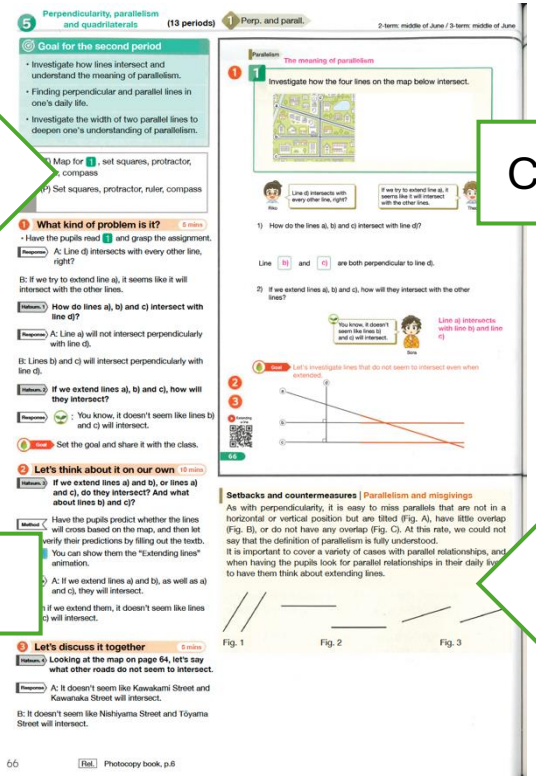
The **bansho book** provides a visual representation of the lesson and gives specific cues (e.g., set the goal on the board).

Specialization

Insights and implications for LS

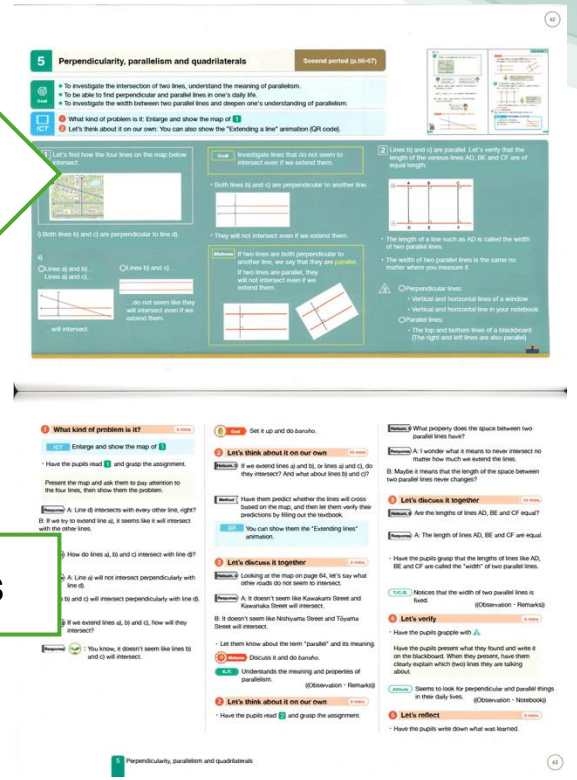


Contextualizes



Implements

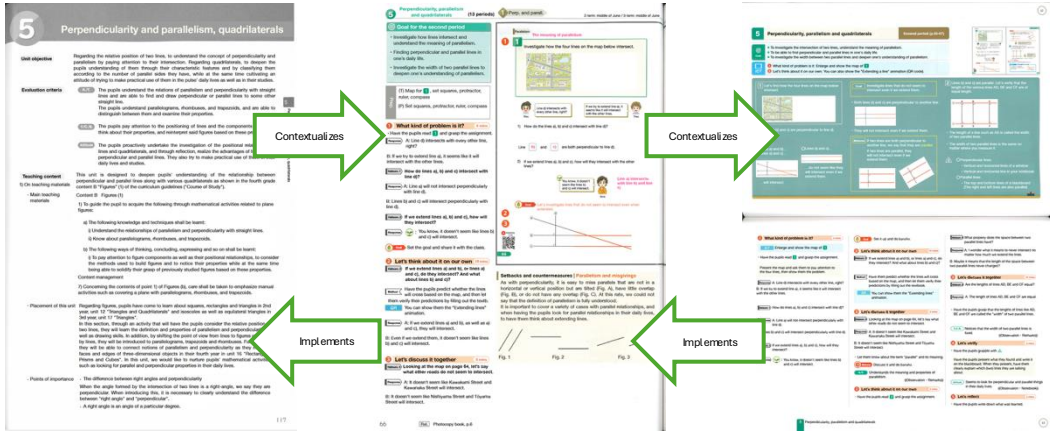
Contextualizes



Implements

Insights and implications for LS

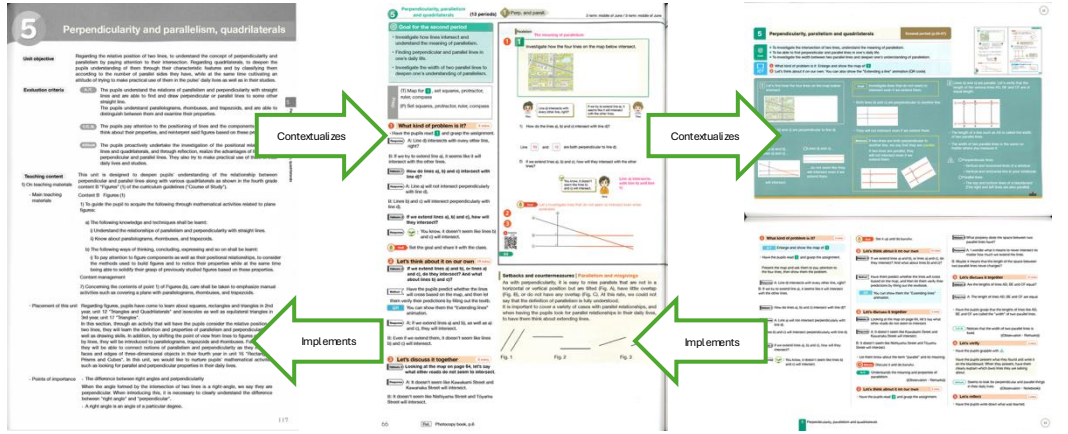
- Distillation process from the research guidebook to the actionable bansho lesson plan.
- The tasks are chosen to fit the context and are linked to the underlying mathematics.
- This has an important effect on *kyouzai kenkyū*.



Global — Local

Insights and implications for LS

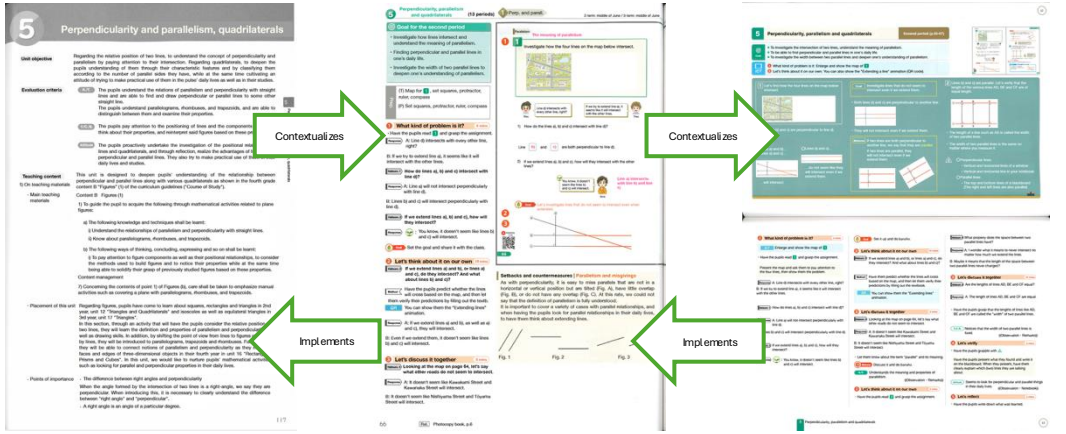
- *kyouzai kenkyū* is “research on instructional materials”. (Watanabe et al., 2008)
- One crucial goal is to “understand what is involved in the planning of a research lesson.” (Watanabe et al., 2008)



Watanabe, Tad & Takahashi, Akihiko & Yoshida, Makoto. (2008). Kyozaikenkyu: A Critical Step for Conducting Effective Lesson Study and Beyond. Inquiry into Mathematics Teacher Education, Association of Mathematics Teacher Educators (AMTE), Monograph Series. 5. 131-142.

Insights and implications for LS

- Global to local structure helps put lessons in a wider context.
- Choices behind the definitions and tasks become apparent.
- This makes *kyouzai kenkyū* more accessible.

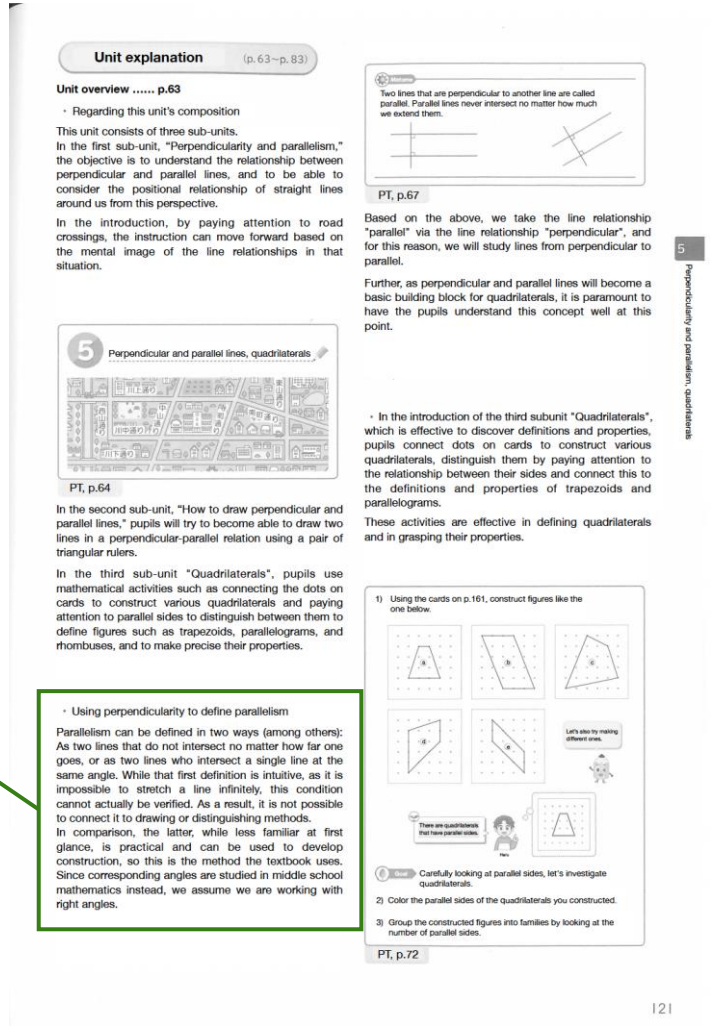


Insights and implications for LS

Making choices apparent

- Using perpendicularity to define parallelism

Parallelism can be defined in two ways (among others): As two lines that do not intersect no matter how far one goes, or as two lines who intersect a single line at the same angle. While that first definition is intuitive, as it is impossible to stretch a line infinitely, this condition cannot actually be verified. As a result, it is not possible to connect it to drawing or distinguishing methods.
In comparison, the latter, while less familiar at first glance, is practical and can be used to develop construction, so this is the method the textbook uses. Since corresponding angles are studied in middle school mathematics instead, we assume we are working with right angles.



Unit explanation (p.63~p.83)

Unit overview p.63

・Regarding this unit's composition

This unit consists of three sub-units.
In the first sub-unit, "Perpendicularity and parallelism," the objective is to understand the relationship between perpendicular and parallel lines, and to be able to consider the positional relationship of straight lines around us from this perspective.

In the introduction, by paying attention to road crossings, the instruction can move forward based on the mental image of the line relationships in that situation.

5 Perpendicular and parallel lines, quadrilaterals

PT, p.64

In the second sub-unit, "How to draw perpendicular and parallel lines," pupils will try to become able to draw two lines in a perpendicular-parallel relation using a pair of triangular rulers.

In the third sub-unit "Quadrilaterals", pupils use mathematical activities such as connecting the dots on cards to construct various quadrilaterals and paying attention to parallel sides to distinguish between them to define figures such as trapezoids, parallelograms, and rhombuses, and to make precise their properties.

・Using perpendicularity to define parallelism

Parallelism can be defined in two ways (among others): As two lines that do not intersect no matter how far one goes, or as two lines who intersect a single line at the same angle. While that first definition is intuitive, as it is impossible to stretch a line infinitely, this condition cannot actually be verified. As a result, it is not possible to connect it to drawing or distinguishing methods.
In comparison, the latter, while less familiar at first glance, is practical and can be used to develop construction, so this is the method the textbook uses. Since corresponding angles are studied in middle school mathematics instead, we assume we are working with right angles.

Two lines that are perpendicular to another line are called parallel. Parallel lines never intersect no matter how much we extend them.

PT, p.67

Based on the above, we take the line relationship "parallel" via the line relationship "perpendicular", and for this reason, we will study lines from perpendicular to parallel.

Further, as perpendicular and parallel lines will become a basic building block for quadrilaterals, it is paramount to have the pupils understand this concept well at this point.

・In the introduction of the third subunit "Quadrilaterals", which is effective to discover definitions and properties, pupils connect dots on cards to construct various quadrilaterals, distinguish them by paying attention to the relationship between their sides and connect this to the definitions and properties of trapezoids and parallelograms.

These activities are effective in defining quadrilaterals and in grasping their properties.

1) Using the cards on p.161, construct figures like the one below.

Carefully looking at parallel sides, let's investigate quadrilaterals.

2) Color the parallel sides of the quadrilaterals you constructed.

3) Group the constructed figures into families by looking at the number of parallel sides.

PT, p.72

5 Perpendicularity and parallelism, quadrilaterals

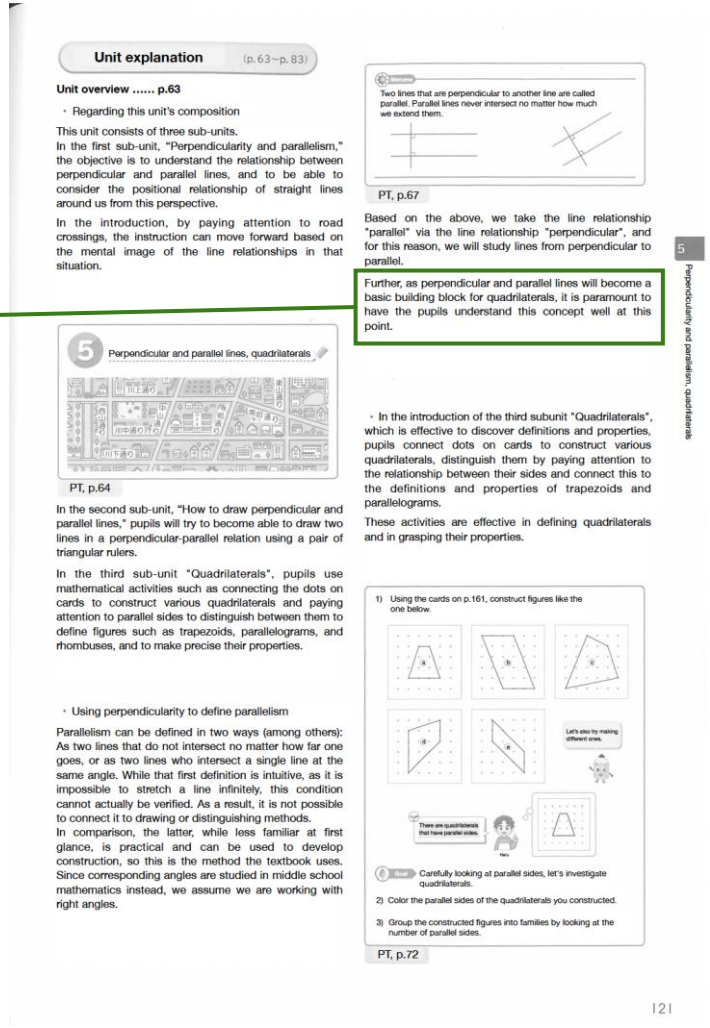
121

Insights and implications for LS

Looking ahead and contextualizing

Based on the above, we take the line relationship "parallel" via the line relationship "perpendicular", and for this reason, we will study lines from perpendicular to parallel.

Further, as perpendicular and parallel lines will become a basic building block for quadrilaterals, it is paramount to have the pupils understand this concept well at this point.



Unit explanation (p.63~p.83)

Unit overview p.63

・Regarding this unit's composition

This unit consists of three sub-units.
In the first sub-unit, "Perpendicularity and parallelism," the objective is to understand the relationship between perpendicular and parallel lines, and to be able to consider the positional relationship of straight lines around us from this perspective.

In the introduction, by paying attention to road crossings, the instruction can move forward based on the mental image of the line relationships in that situation.

5 Perpendicular and parallel lines, quadrilaterals

PT, p.64

In the second sub-unit, "How to draw perpendicular and parallel lines," pupils will try to become able to draw two lines in a perpendicular-parallel relation using a pair of triangular rulers.

In the third sub-unit "Quadrilaterals", pupils use mathematical activities such as connecting the dots on cards to construct various quadrilaterals and paying attention to parallel sides to distinguish between them to define figures such as trapezoids, parallelograms, and rhombuses, and to make precise their properties.

・Using perpendicularity to define parallelism

Parallelism can be defined in two ways (among others): As two lines that do not intersect no matter how far one goes, or as two lines who intersect a single line at the same angle. While that first definition is intuitive, as it is impossible to stretch a line infinitely, this condition cannot actually be verified. As a result, it is not possible to connect it to drawing or distinguishing methods.
In comparison, the latter, while less familiar at first glance, is practical and can be used to develop construction, so this is the method the textbook uses. Since corresponding angles are studied in middle school mathematics instead, we assume we are working with right angles.

Two lines that are perpendicular to another line are called parallel. Parallel lines never intersect no matter how much we extend them.

PT, p.67

Based on the above, we take the line relationship "parallel" via the line relationship "perpendicular", and for this reason, we will study lines from perpendicular to parallel.

Further, as perpendicular and parallel lines will become a basic building block for quadrilaterals, it is paramount to have the pupils understand this concept well at this point.

・In the introduction of the third subunit "Quadrilaterals", which is effective to discover definitions and properties, pupils connect dots on cards to construct various quadrilaterals, distinguish them by paying attention to the relationship between their sides and connect this to the definitions and properties of trapezoids and parallelograms.

These activities are effective in defining quadrilaterals and in grasping their properties.

1) Using the cards on p.161, construct figures like the one below.

Let's also try making different ones.

There are quadrilaterals that have parallel sides.

Hint Carefully looking at parallel sides, let's investigate quadrilaterals.

2) Color the parallel sides of the quadrilaterals you constructed.

3) Group the constructed figures into families by looking at the number of parallel sides.

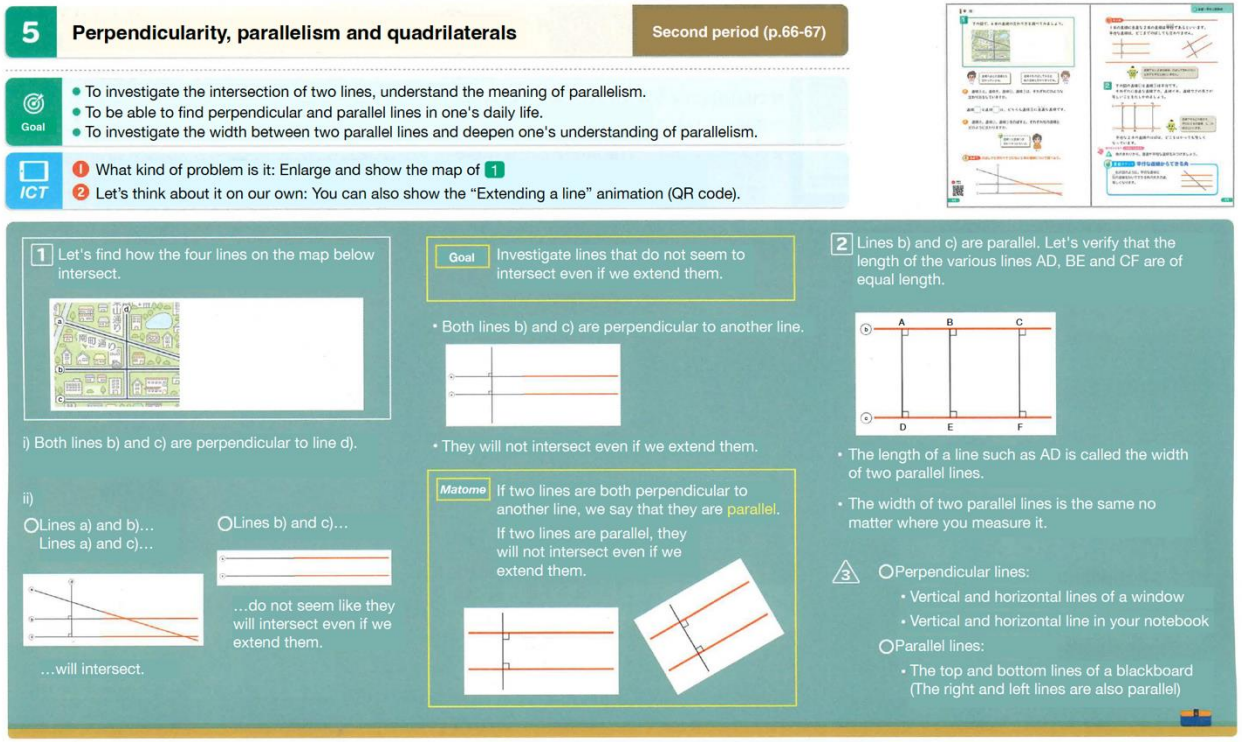
PT, p.72

5 Perpendicularity and parallelism, quadrilaterals

121

Insights and implications for LS

- Another crucial component is the bansho book.
- Bansho is “used to [...] explain the thinking processes and working methods involved in learning tasks”. (Tan, 2021)


5 Perpendicularity, parallelism and quadrilaterals — Second period (p.66-67)

Goal
- To investigate the intersection of two lines, understand the meaning of parallelism.
- To be able to find perpendicular and parallel lines in one's daily life.
- To investigate the width between two parallel lines and deepen one's understanding of parallelism.

ICT
1. What kind of problem is it: Enlarge and show the map of 1
2. Let's think about it on our own: You can also show the “Extending a line” animation (QR code).

1 Let's find how the four lines on the map below intersect.

i) Both lines b) and c) are perpendicular to line d).

ii)
- Lines a) and b)... Lines a) and c)... ...will intersect.
- Lines b) and c)... ...do not seem like they will intersect even if we extend them.

Goal Investigate lines that do not seem to intersect even if we extend them.
- Both lines b) and c) are perpendicular to another line.
- They will not intersect even if we extend them.

Matome If two lines are both perpendicular to another line, we say that they are parallel.
If two lines are parallel, they will not intersect even if we extend them.

2 Lines b) and c) are parallel. Let's verify that the length of the various lines AD, BE and CF are of equal length.
- The length of a line such as AD is called the width of two parallel lines.
- The width of two parallel lines is the same no matter where you measure it.

3
- Perpendicular lines:
  - Vertical and horizontal lines of a window
  - Vertical and horizontal line in your notebook
- Parallel lines:
  - The top and bottom lines of a blackboard (The right and left lines are also parallel)

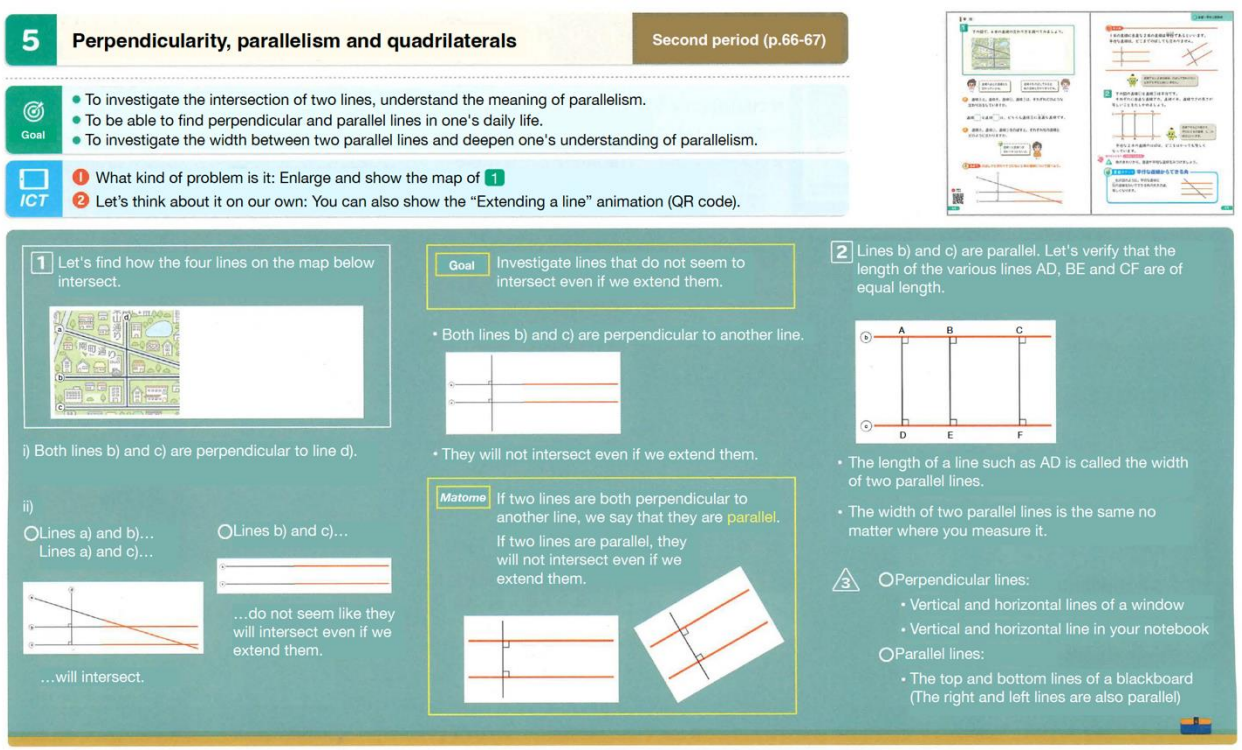
Tan, S. (2021), "Bansho as part of lesson and lesson study: from the origins to the present". *International Journal for Lesson and Learning Studies*, Vol. 10 No. 4 pp. 378–392, doi: https://doi.org/10.1108/IJLLS-09-2021-0076

hep/

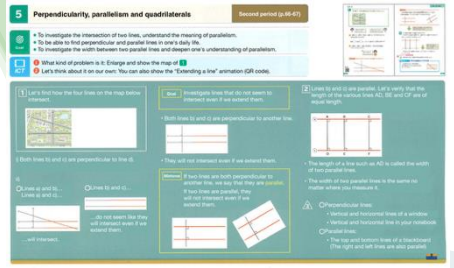
Insights and implications for LS

- Visual representation of the lesson helps teachers prepare the discussion phase (*neriage*).

- The bansho book references the teacher's textbook comments, which references the research guidebook.



Insights and implications for LS



- Prepares teachers to recognize and understand specific student responses.

- In other words, helps teachers **anticipate and notice students' mathematical thinking**.

1 What kind of problem is it? 5 mins

ICT Enlarge and show the map of 1

・Have the pupils read 1 and grasp the assignment.

Present the map and ask them to pay attention to the four lines, then show them the problem.

Response A: Line d) intersects with every other line, right?

B: If we try to extend line a), it seems like it will intersect with the other lines.

Hatsum. 1 How do lines a), b) and c) intersect with line d)?

Response A: Line a) will not intersect perpendicularly with line d).

B: Lines b) and c) will intersect perpendicularly with line d).

Hatsum. 2 If we extend lines a), b) and c), how will they intersect?

Response : You know, it doesn't seem like lines b) and c) will intersect.

Goal Set it up and do *bansho*.

2 Let's think about it on our own 10 mins

Hatsum. 3 If we extend lines a) and b), or lines a) and c), do they intersect? And what about lines b) and c)?

Method Have them predict whether the lines will cross based on the map, and then let them verify their predictions by filling out the textbook.

QR You can show them the "Extending lines" animation.

3 Let's discuss it together 5 mins

Hatsum. 4 Looking at the map on page 64, let's say what other roads do not seem to intersect.

Response A: It doesn't seem like Kawakami Street and Kawanaka Street will intersect.

B: It doesn't seem like Nishiyama Street and Tōyama Street will intersect.

・Let them know about the term "parallel" and its meaning.

Matome Discuss it and do *bansho*.

K./T. Understands the meaning and properties of parallelism.

((Observation・Remarks))

2 Let's think about it on our own 5 mins

・Have the pupils read 2 and grasp the assignment.

Hatsum. 5 What property does the space between two parallel lines have?

Response A: I wonder what it means to never intersect no matter how much we extend the lines.

B: Maybe it means that the length of the space between two parallel lines never changes?

3 Let's discuss it together 10 mins

Hatsum. 6 Are the lengths of lines AD, BE and CF equal?

Response A: The length of lines AD, BE and CF are equal.

・Have the pupils grasp that the lengths of lines like AD, BE and CF are called the "width" of two parallel lines.

T./C./E. Notices that the width of two parallel lines is fixed.

((Observation・Remarks))

4 Let's verify 5 mins

・Have the pupils grapple with A.

Have the pupils present what they found and write it on the blackboard. When they present, have them clearly explain which (two) lines they are talking about.

Attitude Seems to look for perpendicular and parallel things in their daily lives.

((Observation・Notebook))

5 Let's reflect 5 mins

・Have the pupils write down what was learned.

Insights and implications for LS

5 Perpendicularity, parallelism and quadrilaterals — Second period (p.66-67)

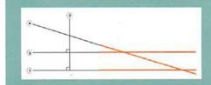
Goal
- To investigate the intersection of two lines, understand the meaning of parallelism.
- To be able to find perpendicular and parallel lines in one's daily life.
- To investigate the width between two parallel lines and deepen one's understanding of parallelism.

ICT
1. What kind of problem is it: Enlarge and show the map of 1
2. Let's think about it on our own: You can also show the "Extending a line" animation (QR code).

1 Let's find how the four lines on the map below intersect.

i) Both lines b) and c) are perpendicular to line d).

ii)
- Lines a) and b)… Lines a) and c)…



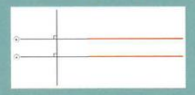
…will intersect.

- Lines b) and c)…

…do not seem like they will intersect even if we extend them.

Goal Investigate lines that do not seem to intersect even if we extend them.

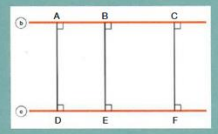
- Both lines b) and c) are perpendicular to another line.



- They will not intersect even if we extend them.

Matome If two lines are both perpendicular to another line, we say that they are parallel.
If two lines are parallel, they will not intersect even if we extend them.

2 Lines b) and c) are parallel. Let's verify that the length of the various lines AD, BE and CF are of equal length.



- The length of a line such as AD is called the width of two parallel lines.
- The width of two parallel lines is the same no matter where you measure it.

3
- Perpendicular lines:
  - Vertical and horizontal lines of a window
  - Vertical and horizontal line in your notebook
- Parallel lines:
  - The top and bottom lines of a blackboard (The right and left lines are also parallel)

1 What kind of problem is it? 5 mins

ICT Enlarge and show the map of 1

- Have the pupils read 1 and grasp the assignment.

Present the map and ask them to pay attention to the four lines, then show them the problem.

Response A: Line d) intersects with every other line, right?

B: If we try to extend line a), it seems like it will intersect with the other lines.

Hatsum. 1 How do lines a), b) and c) intersect with line d)?

Response A: Line a) will not intersect perpendicularly with line d).

B: Lines b) and c) will intersect perpendicularly with line d).

Hatsum. 2 If we extend lines a), b) and c), how will they intersect?

Response : You know, it doesn't seem like lines b) and c) will intersect.

Goal Set it up and do *bansho*.

2 Let's think about it on our own 10 mins

Hatsum. 3 If we extend lines a) and b), or lines a) and c), do they intersect? And what about lines b) and c)?

Method Have them predict whether the lines will cross based on the map, and then let them verify their predictions by filling out the textbook.

QR You can show them the "Extending lines" animation.

3 Let's discuss it together 5 mins

Hatsum. 4 Looking at the map on page 64, let's say what other roads do not seem to intersect.

Response A: It doesn't seem like Kawakami Street and Kawanaka Street will intersect.

B: It doesn't seem like Nishiyama Street and Tōyama Street will intersect.

- Let them know about the term "parallel" and its meaning.

Matome Discuss it and do *bansho*.

K./T. Understands the meaning and properties of parallelism.
((Observation・Remarks))

2 Let's think about it on our own 5 mins

- Have the pupils read 2 and grasp the assignment.

Hatsum. 5 What property does the space between two parallel lines have?

Response A: I wonder what it means to never intersect no matter how much we extend the lines.

B: Maybe it means that the length of the space between two parallel lines never changes?

3 Let's discuss it together 10 mins

Hatsum. 6 Are the lengths of lines AD, BE and CF equal?

Response A: The length of lines AD, BE and CF are equal.

- Have the pupils grasp that the lengths of lines like AD, BE and CF are called the "width" of two parallel lines.

T./C./E. Notices that the width of two parallel lines is fixed.
((Observation・Remarks))

4 Let's verify 5 mins

- Have the pupils grapple with

Have the pupils present what they found and write it on the blackboard. When they present, have them clearly explain which (two) lines they are talking about.

Attitude Seems to look for perpendicular and parallel things in their daily lives.
((Observation・Notebook))

5 Let's reflect 5 mins

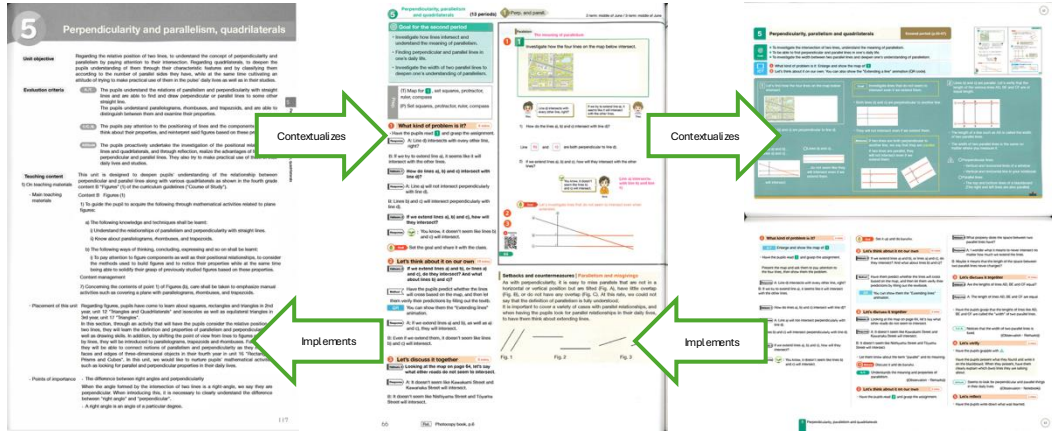
- Have the pupils write down what was learned.

Effectively, a lesson plan…
built on the other books.

Conclusion

Conclusion

- Global-local structure with **mathematics front and center**.
- Planning and *kyouzai kenkyū* **become more accessible**.
- Integrated approach **helps teachers anticipate students' mathematical thinking** better.



References

Watanabe, Tad & Takahashi, Akihiko & Yoshida, Makoto. (2008). Kyozaikenkyu: A Critical Step for Conducting Effective Lesson Study and Beyond. Inquiry into Mathematics Teacher Education, Association of Mathematics Teacher Educators (AMTE), Monograph Series. 5. 131-142.

Tan S (2021), "Bansho as part of lesson and lesson study: from the origins to the present". *International Journal for Lesson and Learning Studies*, Vol. 10 No. 4 pp. 378–392, doi: https://doi.org/10.1108/IJLLS-09-2021-0076